1.5 Renormalization Scale Dependence

this has been checked!

If the perturbative coefficients c of $\alpha_s(\mu_r)$ have been computed for a renormalization scale $\mu_r = \mu_0$, the value of the Observable O at this scale is given by

$$O(\mu_0) = a^n(\mu_0) \cdot c_{\mathrm{LO}} \, + \, a^{n+1}(\mu_0) \cdot c_{\mathrm{NLO}}(\mu_0) \, + \, a^{n+2}(\mu_0) \cdot c_{\mathrm{NNLO}}(\mu_0) \, , \tag{1.12}$$

with $a = \alpha_s/2\pi$ and n is the power of α_s in the LO term. We assume that the coefficients c have already been integrated over the PDFs and summed over all relevant partonic subprocesses. The prediction for the observable at any other scale $\mu_r = x_\mu \cdot \mu_0$ (this defines x_μ as $x_\mu = \mu_r/\mu_0$) can be computed as

$$\begin{aligned} O(\mu_r) = \quad & a^n(\mu_r) \cdot c_{\mathrm{LO}} \\ & + a^{n+1}(\mu_r) \cdot [c_{\mathrm{NLO}}(\mu_0) + n\beta_0 L \, c_{\mathrm{LO}}] \\ & + a^{n+2}(\mu_r) \cdot [c_{\mathrm{NNLO}}(\mu_0) \, + \, (n+1)\beta_0 L \, c_{\mathrm{NLO}}(\mu_0) \\ & \qquad + \left(\frac{n(n+1)}{2} \beta_0^2 L^2 + n \frac{\beta_1}{2} L \right) c_{\mathrm{LO}}] \end{aligned} \tag{1.13}$$

with $a = 4\pi\alpha_s$, $L = \ln(x_\mu)$, $C_A = 3$,$C_F = 4/3$ (and $n_f = 5$ for jet production at the Tevatron) and

$$\begin{aligned} \beta_0 &= (11 C_A - 2 n_f)/3 \, , \\ \beta_1 &= 34 C_A^2/3 - 2 n_f (C_F + 5 C_A/3) \, . \end{aligned}$$

Attention: This works only for full orders – not for partial orders as e.g. the $\mathcal{O}(\alpha_s^4)$ (or “2-loop”) contributions from threshold corrections.

Up to NLO this can be rewritten as

$$\begin{aligned} O(\mu_r) = \quad & a^n(\mu_r) \cdot c_{\mathrm{LO}} \\ & + a^{n+1}(\mu_r) \cdot c_{\mathrm{NLO}}(\mu_0) \\ & + a(\mu_r) \cdot (n\beta_0 L) \cdot [a^n(\mu_r) \cdot c_{\mathrm{LO}}] \; , \end{aligned} \tag{1.14}$$

however, we include the full expression in fastNLO v2.

In other words, when choosing μ_r different from the value μ_0 for which the table has been computed, the following calculations are required in a NLO calculation:

1. The computation of the LO contribution, however accessing α_s not at μ_0, but at μ_r
2. The computation of the NLO contribution, again accessing α_s at μ_r
3. One additional calculation of a “modified” LO contribution in which the coefficients are multiplied with $(n\beta_0 L)$ and α_s is accessed one order higher than in (1).

Correspondingly, the calculation of a NNLO cross section requires additional modified calculations of the NLO and LO terms.

In the fastNLO v2 Fortran user code this is implemented in the routines FT9999PM and FT9999MT. The routine FT9999MT multiplies the table elements with the PDFs and α_s (in the appropriate power). To enable the a-posteriori μ_r variation, this routine has two additional arguments. The variable $Iaddpow$ defines by how many additional powers α_s is computed and $factor$ specifies an additional factor by which this contribution is multiplied. When computing the modified LO contribution discussed above, one would set $Iaddpow = 1$ and $factor = (n\beta_0 L)$.